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Note

Enhanced Boundary Pressure Update for Incompressible Flow Simulation

Incompressible flow simulation is frequently performed using a primitive variable (u, v, p) formulation of the governing equations. These include a momentum equation in each coordinate direction and a pressure Poisson equation. The pressure Poisson equation incorporates mass conservation and assists in decoupling the pressure from the velocity during an iterative lagging procedure. The Poisson equation is elliptic in nature and requires specification of boundary pressures over each boundary in the flow domain. However, the pressure distribution on the surface of bodies in the flow is not known a priori and is sought as part of the problem solution. Several techniques have been proposed for the update of boundary pressure values as the simulation progresses toward satisfaction of mass conservation (zero dilatation). Two of these are the method of Viecelli [1, 2], and the Raithby–Schneider PUMPIN technique [3]. In a recent hydrodynamic investigation performed in body-fitted coordinates [4], a modification of the method of Viecelli was developed and compared with the two aforementioned methods. The modification resulted in a significant acceleration of the boundary pressure update process, enabling extended iteration for reduction of the dilatation residual used as the measure of incompressibility of the flow.

Briefly, the method of Viecelli updates the boundary pressures in response to the dilatation at each boundary point. The dilatation Di defined in Eq. 1 is zero in an incompressible flow, and is held as small as possible in a numerical approximation to incompressible flow,

$$\mathrm{Di} = u_x + v_y. \tag{1}$$

If the dilatation is positive at a boundary point, the pressure is reduced to prevent expanding fluid from crossing the solid boundary. If the dilatation is negative, the pressure is increased to prevent separation from the boundary. A zero dilatation value requires no pressure correction. Viecelli offers the following relation for update of the pressure from the k to the $k+1$ iteration level,

$$p^{k+1} = p^k - \varepsilon \frac{\rho \delta^2 \,\mathrm{Di}}{2\,\Delta t}, \tag{2}$$

where ρ is the fluid density, δ is the cell spacing, Δt is the timestep, and ε is a parameter held less than unity to maintain stability. The optimum value of the

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parameter ε is found by trial and error. In many current applications, a body-fitted coordinate transformation is used, resulting in nonuniform cell spacing [5]. Thompson and Shanks [6] cite work by Hodge to transform the Viecelli pressure update to body-fitted coordinates in two dimensions. After nondimensionalization and transformation to the curvilinear coordinates (ξ, η), Eq. (2) is represented as

$$p^{k+1} = p^k - \frac{2\varepsilon J^2 \tilde{\mathrm{D}}\mathrm{i}}{(\alpha+\gamma)\,\Delta t}, \tag{3}$$

where

$$J = x_\xi y_\eta - x_\eta y_\xi \tag{4}$$

$$\alpha = x_\eta^2 + y_\eta^2 \tag{5}$$

$$\gamma = x_\xi^2 + y_\xi^2 \tag{6}$$

$$\tilde{\mathrm{D}}\mathrm{i} = \frac{1}{J}\left[u_\xi y_\eta - u_\eta y_\xi - v_\xi x_\eta + v_\eta x_\xi\right]. \tag{7}$$

Under a body-fitted coordinate transformation, uniform cell spacing in the computational plane seldom leads to a uniform spacing in the physical plane. Mesh generation techniques emphasize the concentration of grid points in regions of large gradients and an increase in grid point spacing in regions of small anticipated change. In Eq. (3), the update process is dependent upon the values of the Jacobian determinant J, as well as the metric coefficients α and γ at each boundary grid point in the flow domain. The square of the cell spacing determines the rate at which the update process converges toward a zero dilatation flow. If the value of the cell spacing is very small, as required in regions of large gradients, the update process will require an extensive amount of computational effort. Also, local variations in grid point spacing can be viewed as effectively varying the parameter ε in comparison to a uniformly-spaced mesh. Thus a single optimum value for the parameter ε cannot be determined.

The detail of a computational mesh generated between a cylinder and a plane boundary is shown in Fig. 1. Following satisfactory evaluation of the truncation error as a function of mesh skewness and grid point density, the simulation of low Reynolds number flow using the Viecelli update technique was found to require in excess of 90 cpu s per time step to achieve a nondimensional dilatation of less than 10^{-2}. These figures are given for a NAS/9000 mainframe system where both the cpu time requirement, and consequently, the accuracy, were unsatisfactory for a dynamic simulation involving many time steps.

The convergence of the Viecelli boundary pressure update method toward a zero dilatation approximation was examined in detail for the two-boundary configuration shown in Fig. 1. It was evident that the wide range of cell spacing about the cylinder and at the plane boundary led to rapid resolution in areas of large cell spacing (large J^2 value) while others with small cell spacing were making negligible

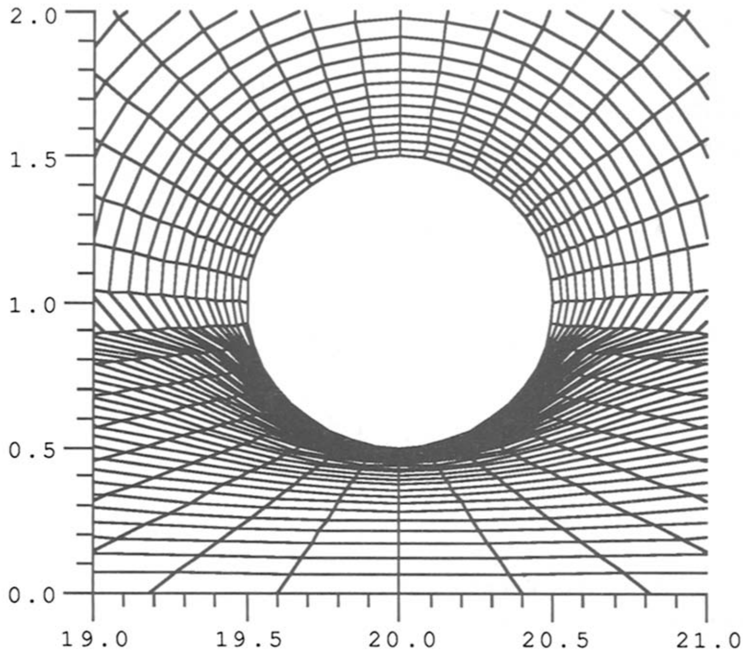

FIG. 1. Computational mesh for cylinder approaching a plane boundary. Ordinate and abscissa are scaled to cylinder diameter units.

progress toward an incompressible flow solution. Simply increasing the parameter ε had the same destabilizing effect in nonuniform spacing that Viecelli's analysis predicts for uniform spacing. However, the modification developed in [4] involved use of the values of J, α, and γ computed for the largest cell spacing on the cylinder in calculations over the entire cylinder boundary and was found to enhance the speed of the update process while maintaining stability over the entire range of mesh aspect ratios investigated. By choosing the coefficients of the transformed dilatation in Eq. (3) in this manner, an *approach* to an effective optimum value of the parameter ε is achieved. However, as noted above, the nonuniformity of mesh spacing under the subject body-fitted coordinate transformation leads to a large number of local optimum values. Further examination of the problem may lead to alternate approaches to accelerating this boundary pressure update technique. Stability was maintained as long as values for the largest cell on one boundary remained smaller than the smallest cell on the opposite boundary with which it was in communication. In short, the update process for two boundaries having nonuniform cell spacing was found to remain stable as long as one boundary was allowed to converge more rapidly than the other. This conclusion resulted in reduction of the cpu time requirement from 90 to 2.94s per time step while improving the zero dilatation approximation from 10^{-2} to 10^{-5}.

One additional method examined was the pressure update through multiple path integration (PUMPIN) technique of Raithby and Schneider which was transformed to body-fitted coordinates and substituted for the Viecelli method at the plane boundary. The PUMPIN technique is based on the fact that the Navier–Stokes equations hold at all points in the flow field. The pressure distribution along a boundary is obtained by solution of a momentum equation for the pressure gradient using the current velocity field. The gradient is numerically integrated from a known initial value such as a free stream pressure. Because the current velocity field is not necessarily incompressible, the pressure at a boundary point achieved by integration along one path will not necessarily equal that obtained by integration along an alternate path. However, the error can be averaged over multiple paths and reduced in an iterative process. The PUMPIN technique would be a fast and effective alternative to the Viecelli update method for problems having a single boundary. In the cylinder and plane boundary application, it was necessary to use the Viecelli update at one of the boundaries to generate an initial pressure value for the PUMPIN technique at the other boundary. This offered no net improvement over the use of the Viecelli method at both boundaries.

In summary, the boundary pressure update using the Viecelli method in multiple boundary problems was enhanced through selection of grid metric parameters that permit a more uniform adjustment of the boundary pressure field in body-fitted coordinates. Care must still be taken to assure the stability of the technique in accordance with the observations provided above.

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